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CONTENTS

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Journal of Mathematical Sciences, Vol. 274, No. 2, August, 2023

TO MEMORY OF MAKHMUD S. SALAKHITDINOV

Makhmud S. Salakhitdinov (1933–2018) *had a strong influence on the development of science and education in Uzbekistan, being Director of the Institute of Mathematics* (1967–1985), *Minister of Higher and Special Secondary Education* (1985–1988), *and President of the Academy of Sciences* (1988–1994) *of the Republic of Uzbekistan. At the same time, his personal contribution to mathematical sciences is also very significant and influential. The special issue* 274 (2) *of JMS presents current results on local and nonlocal boundary value problems for mixed type equations, integral equations of fractional order, and other topics close to the research of M. S. Salakhitdinov.*

Makhmud Salakhitdinovich Salakhitdinov
November 23, 1933 – April 27, 2018

Makhmud Salakhitdinovich Salakhitdinov was born in the city of Namangan in eastern Uzbekistan on November 23, 1933. In 1950, M.S. entered the Central Asian State University (now, the National University of Uzbekistan named after Mirzo Ulugbek) and graduated with honors from the Faculty of Physics and Mathematics (1955) and postgraduate studies (1958). He received

International Mathematical Schools. Vol. 3. *Mathematical Schools in Uzbekistan. In Memory of M. S. Salakhitdinov*

Journal of Mathematical Sciences, Vol. 274, No. 2, August, 2023

DETERMINATION OF TEMPERATURE AT THE OUTER BOUNDARY OF A BODY

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We study the problem of finding the temperature on the outer boundary of a cylindrical domain under given conditions on the inner boundary. We establish the existence and uniqueness of a solution. Bibliography: 13 *titles.*

1 Introduction

1.1. In this paper, we establish the solvability of the boundary value problem associated with the following physical model. Suppose that a rod radiating heat is placed in a cylindrical inhomogeneous domain $\Omega \times \mathbb{R}$. The temperature on the rod surface and the heat flow through the rod surface are known. It is required to determine the temperature at the outer boundary of the cylindrical domain.

We assume that Ω is a two-dimensional convex bounded domain, x^0 is an arbitrary point of Ω, and $\rho < \operatorname{dist}\{x^0, \partial\Omega\}$. We set

$$\Omega_\rho = \{(x_1, x_2) \in \Omega \mid (x_1 - x_1^0)^2 + (x_2 - x_2^0)^2 > \rho^2\}.$$

It is clear that $\partial\Omega_\rho = \Gamma_\rho \cup \partial\Omega$, where

$$\Gamma_\rho = \{(x_1, x_2) \in \mathbb{R}^2 \mid (x_1 - x_1^0)^2 + (x_2 - x_2^0)^2 = \rho^2\}.$$

Consider the process of heating the cylindrical domain

$$D = \Omega_\rho \times \mathbb{R} = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid (x_1, x_2) \in \Omega_\rho, \ -\infty < x_3 < +\infty\}.$$

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International Mathematical Schools. Vol. 3. *Mathematical Schools in Uzbekistan. In Memory of M. S. Salakhitdinov*

Journal of Mathematical Sciences, Vol. 274, No. 2, August, 2023

INVERSE PROBLEM OF BITSADZE–SAMARSKII TYPE FOR A TWO-DIMENSIONAL PARABOLIC EQUATION OF FRACTIONAL ORDER

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We consider a nonlocal inverse problem of Bitsadze–Samarskii type for a degenerate fractional order parabolic equation with the Gerasimov–Caputo operator in two spatial variables. The problem is reduced to the study of a spectral boundary value problem for a second order ordinary differential equation with respect to the spatial variable. We study spectral properties of the obtained and adjoint problems. We find eigenvalues with the corresponding root functions and establish their basis property. We prove the uniqueness and existence theorems amd construct the solution in the form of an absolutely and uniformly convergent biorthogonal series. Bibliography: 22 *titles.*

1 Introduction and Statement of the Problem

Mathematical modeling of many processes occurring in the real world leads to problems of determining the coefficients or right-hand side of a differential equation by using known data from its solution. Such problems are called inverse problems of mathematical physics. The study of inverse problems is of vital interest for many areas of science and engineering such

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International Mathematical Schools. Vol. 3. *Mathematical Schools in Uzbekistan. In Memory of M. S. Salakhitdinov*

Journal of Mathematical Sciences, Vol. 274, No. 2, August, 2023

LINEAR INVERSE PROBLEM FOR 3-DIMENSIONAL CHAPLYGIN EQUATION WITH SEMI–NONLOCAL BOUNDARY CONDITIONS IN A PRISMATIC UNBOUNDED DOMAIN

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The paper addresses the issues of well-posedness of the linear inverse problem for the three-dimensional Chaplygin equation in a prismatic unbounded domain with semi-local boundary conditions. Using ε-regularization methods, a priori estimates, and a sequence of approximations with application of the Fourier transform, we establish the existence and uniqueness of a generalized solution to the problem in some class of integrable functions. Bibliography: 25 *titles.*

1 Introduction and Statement of the Problem

In the process of studying nonlocal problems, a close connection between problems with nonlocal boundary conditions and inverse problems was identified. Inverse problems for classical equations of parabolic, elliptic, and hyperbolic type are sufficiently well studied [1]–[5]. Mixed type equations of the first and second kind in bounded domains were considered in [6]–[13]. Direct and inverse problems for mixed type equations of the first and second kind in unbounded domains have been studied much less [14]–[17]. We attempt to partially fill this gap in this work.

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Journal of Mathematical Sciences, Vol. 274, No. 2, August, 2023

CONDITIONAL WELL-POSEDNESS OF THE INITIAL-BOUNDARY VALUE PROBLEM FOR A SYSTEM OF INHOMOGENEOUS MIXED TYPE EQUATIONS WITH TWO DEGENERATION LINES

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We study the conditional well-posedness of the initial-boundary value problem for a system of inhomogeneous mixed type equations with two degeneration lines. We establish the conditional well-posedness of the problem, i.e., we prove the uniqueness and conditional stability theorems. Bibliography: 25 *titles.*

1 Introduction

In this paper, we study the initial-boundary value problem for a system of inhomogeneous mixed-type partial differential equations of the second order with two degeneration lines.

The theory of boundary value problems for mixed type equations is intensively developed due to its numerous applications, in particular, in gas dynamics, magnetohydrodynamics, infinitesimal bendings of surfaces, shell theory, predicting the level of groundwater (see, for example, [1]–[3]).

Well-posed boundary value problems for mixed type equations were studied in [4]–[7] (see the references therein). The case of two degeneration lines was considered in [8].

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International Mathematical Schools. Vol. 3. *Mathematical Schools in Uzbekistan. In Memory of M. S. Salakhitdinov*
1072-3374/23/2742-0201

Journal of Mathematical Sciences, Vol. 274, No. 2, August, 2023

INFINITE SUMMATION FORMULAS FOR TRIPLE LAURICELLA HYPERGEOMETRIC FUNCTIONS

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Inspired by the work by Brychkov and Saad who gave infinite summation formulas for Appell functions (of two variables) with the help of summation theorems, we, using a new inverse pair of symbolic operators, establish infinite summation formulas for four Lauricella functions of three variables. As an application of the results obtained, we present integral representation formulas. Bibliography: 30 titles.

1 Introduction and Definitions

Hypergeometric functions of one and more variables naturally occur in a wide variety of problems in applied mathematics, statistics, operations research, theoretical physics, and engineering sciences. For instance, Srivastava and Kashyap [1] presented a number of interesting applications of hypergeometric functions of one and more variables in queuing theory and related stochastic processes. The work of Niukkanen [2] on multiple hypergeometric functions was motivated by various physical and quantum chemical applications of such functions. Especially, many problems in gas dynamics lead to solutions of degenerate second order partial differential equations which are then solvable in terms of multiple hypergeometric functions. Among examples, we can cite the problem of adiabatic flat-parallel gas flow without whirlwind, the flow problem of supersonic current from vessel with flat walls, and a number of other problems connected with gas flow [3].

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International Mathematical Schools. Vol. 3. *Mathematical Schools in Uzbekistan. In Memory of M. S. Salakhitdinov*
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Journal of Mathematical Sciences, Vol. 274, No. 2, August, 2023

INITIAL-BOUNDARY VALUE PROBLEMS WITH GENERALIZED SAMARSKII–IONKIN CONDITION FOR PARABOLIC EQUATIONS WITH ARBITRARY EVOLUTION DIRECTION

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We study the solvability of boundary value problems nonlocal with respect to the spatial variable with the generalized Samarskii–Ionkin condition for parabolic equations

$$h(t)u_t - \frac{\partial}{\partial x}(a(x)u_x) + c(x,t)u = f(x,t),$$

where $x \in (0,1)$, $t \in (0,T)$ and $h(t)$, $a(x)$, $c(x,t)$, $f(x,t)$ are given functions. If $a(x)$ is positive, then the function $h(t)$ can have different signs at different points of $[0,T]$ or even vanish on a set of positive measure in $[0,T]$. We prove the existence and uniqueness of regular solutions, i.e., solutions possessing all weak derivatives (in the sense of Sobolev) occurring in the corresponding equation. The obtained results are new even for the classical Samarskii–Ionkin problem for the heat equation. Bibliography: 21 *titles.*

1 Introduction

The paper is devoted to the study of the solvability of spatially nonlocal boundary value problems for differential equations

$$h(t)u_t - \frac{\partial}{\partial x}(a(x)u_x) + c(x,t)u = f(x,t), \tag{1.1}$$

where $a(x)$ is a positive function and the function $h(t)$ is not of a fixed sign. In the literature, such equations are called *parabolic equations with varying evolution direction* (specify that for different t the function $h(t)$ can take either positive, or negative, or even zero values, in particular, on a set of measure zero). The solvability of various local boundary value problems for such

International Mathematical Schools. Vol. 3. *Mathematical Schools in Uzbekistan. In Memory of M. S. Salakhitdinov*

Journal of Mathematical Sciences, Vol. 274, No. 2, August, 2023

BOUNDARY VALUE PROBLEM FOR AN ODD ORDER EQUATION WITH MULTIPLE CHARACTERISTICS

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We prove the unique solvability of the nonlinear boundary value problem for an odd order nonlinear equation with multiple characteristics in a curvilinear domain. The uniqueness of a solution is established by the method of energy integrals by using some elementary inequalities and Friedrichs type inequalities. To prove the existence of a solution to this problem, an auxiliary problem is considered whose Green function is constructed. With the help of this auxiliary problem, the original problem is reduced to a system of Hammerstein integral equations. The solvability of the nonlinear system is proved by the contraction mapping principle. Bibliography: 8 titles.

1 Introduction

We study the odd order equations

$$\text{(MC)} \qquad L(u) \equiv \frac{\partial^{2n+1}u}{\partial x^{2n+1}} + (-1)^n \frac{\partial u}{\partial y} = f\Big(x, y, u(x,y), \frac{\partial u}{\partial x}, \dots, \frac{\partial^{2n}u}{\partial x^{2n}}\Big),$$

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International Mathematical Schools. Vol. 3. *Mathematical Schools in Uzbekistan. In Memory of M. S. Salakhitdinov*

Journal of Mathematical Sciences, Vol. 274, No. 2, August, 2023

INVERSE PROBLEMS OF RECOVERING THE HEAT TRANSFER COEFFICIENT WITH INTEGRAL DATA

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We consider the inverse problem of recovering the heat transfer coefficient represented as a finite segment of the Fourier series with coefficients depending on time. The overdetermination data are taken in the form of integrals of a solution with weights over a space domain. We prove that a solution to the problem is uniquely determined and continuously depends on the data. Bibliography: 16 *titles.*

1 Introduction

We study the parabolic equation

$$Lu = u_t - \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} a_{ij}(t,x) u_{x_j} + \sum_{i=1}^{n} a_i(t,x) u_{x_i} + a_0(t,x) u = f, \tag{1.1}$$

where $G \subset \mathbb{R}^n$ is a bounded domain with boundary Γ of class C^2 (see the definitions in [1, Chapter 17]), $t \in (0,T)$. We set $Q = (0,T) \times G$ and $S = (0,T) \times \Gamma$. We consider Equation (1.1) with boundary and initial conditions

$$B(t,x)u|_S = \frac{\partial u}{\partial N} + \sigma(t,x)u\Big|_S = g, \quad u|_{t=0} = u_0(x), \tag{1.2}$$

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Journal of Mathematical Sciences, Vol. 274, No. 2, August, 2023

D'ALEMBERT FORMULA FOR POROELASTICITY SYSTEM DESCRIBED BY THREE ELASTIC PARAMETERS

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We consider the Cauchy problem for a one-dimensional homogeneous system of poroelasticity equations described by three elastic parameters in a reversible hydrodynamic approximation. We find a solution to the Cauchy problem in the form of the d'Alembert formula and show the influence of porosity on the acoustic wave propagation. Bibliography: 5 *titles.*

1 Introduction

For applied problems of the theory of elastic wave propagation it is often required to take into account the medium porosity, fluid saturation of the medium, and the hydrodynamic background. Similar questions arise in seismology (see [1]–[3] and the references therein). The nonlinear mathematical model for a fluid-saturated porous elastically deformable medium constructed in [4] is based on three main principles: the conservation laws, the Galileo principle of relativity, and the consistency of the equations of saturating fluid motion with thermodynamic equilibrium

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\mathbf{j} = 0, \quad \frac{\partial S}{\partial t} + \mathrm{div}\Big(\frac{S}{\rho}\mathbf{j}\Big) = 0, \quad \mathbf{j} = \rho_s \mathbf{u}_1 + \rho_l \mathbf{u}_2,$$

$$\frac{\partial \rho_l}{\partial t} + div(\rho_l \mathbf{u}_2) = 0, \quad \frac{\partial g_{ik}}{\partial t} + g_{kj}\partial_i u_{1j} + g_{ij}\partial_k u_{1j} + u_{1j}\partial_j g_{ik} = 0,$$

$$\frac{\partial e}{\partial t} + \mathrm{div}\mathbf{Q} = 0, \quad \rho_s = \mathrm{const}\ \sqrt{\det(g_{ik})},$$

$$\frac{\partial j_i}{\partial t} + \partial_k(\rho_s u_{1i}u_{1k} + \rho_s u_{2i}u_{2k} + p\delta_{ik} + h_{ij}g_{jk}) = 0,$$

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International Mathematical Schools. Vol. 3. *Mathematical Schools in Uzbekistan. In Memory of M. S. Salakhitdinov*

Journal of Mathematical Sciences, Vol. 274, No. 2, August, 2023

NONLOCAL BOUNDARY VALUE PROBLEM FOR A MIXED TYPE EQUATION WITH FRACTIONAL PARTIAL DERIVATIVE

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We study the nonlocal boundary value problem for a mixed type equation with the Riemann–Liouville fractional partial derivative. In the hyperbolic part of the domain, the functional equation is solved by the iteration method. The problem is reduced to solving a fractional differential equation. Bibliography: 13 *titles.*

Fractional partial differential equations can arise in the mathematical modeling of physical media with fractal geometry [1]. Boundary value problems for the fractional diffusion equation were studied in [2]–[4]. A certain family of generalized Riemann-Liouville fractional derivative operators $D^{\alpha,\beta}_{a+}$ of order α and β was considered in [5]. Applications of such operators can be found in [6]. Fractional differential equations arise, in particular, in problems of classical mechanics, hydrodynamics, heat conduction, diffusion, in the study of physical processes of stochastic transfer, in the use of the concept of a fractal in condensed matter physics. The need to study boundary value problems for fractional differential equations is caused by the fact that many problems in the theory of fluid filtration in a fractured medium with fractal fracture geometry lead to fractional differential equations. Fractional derivatives are also used to describe physical processes of stochastic transfer [7]. The fractional order diffusion was discussed in [8]. The problem for an equation involving the Riemann–Liouville fractional partial derivative with a boundary condition containing a fractional integro-differentiation operator was studied in [9]. In the present paper, we consider the nonlocal boundary value problem for a mixed type equation with fractional partial derivative.

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International Mathematical Schools. Vol. 3. *Mathematical Schools in Uzbekistan. In Memory of M. S. Salakhitdinov*

Journal of Mathematical Sciences, Vol. 274, No. 2, August, 2023

THE DIRICHLET PROBLEM FOR AN ELLIPTIC EQUATION WITH THREE SINGULAR COEFFICIENTS AND NEGATIVE PARAMETERS

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We are interested in regular solutions to the first boundary value problem for a three-dimensional elliptic type equation with three singular coefficients and negative parameters in a rectangular parallelepiped. To study the problem, we use methods of spectral analysis. The solution is constructed in the form of the double Fourier–Bessel series. Bibliography: 14 *titles.*

1 Introduction. Statement of the Problem

The study of stationary physical processes usually leads to boundary problems for elliptic type equations. The theory of such problems has a rich history and is one of the most rapidly developing parts of the theory of partial differential equations. We give a brief historical background on boundary value problems for multidimensional elliptic type equations with singular coefficients. In 1937, Agostinelli [1] considered the Dirichlet problem for a three-dimensional elliptic equation with one singular coefficient in a half-space. In 1949, Olevsky [2] announced an explicit formula for the solution to the Dirichlet problem for the equation

$$\Delta u + (p/x_n)u_{x_n} = f$$

in a multidimensional half-ball, where f is a given function and Δ is the n dimensional Laplace operator. This problem for the equation

$$\Delta u + (a/x_n)u_{x_n} - b^2 u = 0, \quad a < 1,$$

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Journal of Mathematical Sciences, Vol. 274, No. 2, August, 2023

BITSADZE–SAMARSKII TYPE PROBLEM FOR A MIXED TYPE EQUATION THAT IS ELLIPTIC IN THE FIRST QUADRANT OF THE PLANE

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We consider the problem of Bitsadze–Samarskii type for a generalized Tricomi equation with a spectral parameter in the case where the equation is elliptic in the first quadrant of the plane. We establish the existence and uniqueness of a solution to the problem. Bibliography: 5 *titles.*

1 Statement of the Problem

We consider the equation

$$\operatorname{sgn} y|y|^m u_{xx} + u_{yy} - \lambda^2 |y|^m u = 0 \tag{1.1}$$

in the unbounded domain $\Omega = \Omega_1 \cup l_1 \cup \Omega_2$, where $\Omega_1 = \{(x,y) : x > 0, y > 0\}$ and Ω_2 is a domain in the half-plane $y < 0$ bounded by the segment AB of the straight line $y = 0$ and the characteristics

$$\mathrm{AC}: \ x - [2/(m+2)](-y)^{(m+2)/2} = 0,$$

$$\mathrm{BC}: \ x + [2/(m+2)](-y)^{(m+2)/2} = 1$$

of Equation (1.1) outgoing from the points $A(0,0)$ and $B(1,0)$. We set $\beta = m/(2m+4)$, $l_1 = \{(x,y) : 0 < x < 1, y = 0\}$, $l_2 = \{(x,y) : x > 1, y = 0\}$, $l_3 = \{(x,y) : y > 0, x = 0\}$, $\theta_0(x) = (x/2, -(m+2)/2 \cdot x/2]^{\frac{2}{m+2}})$ is the point of intersection of the characteristic of Equation (1.1) outgoing from the point $(x,0) \in l_1$ with the characteristic AC. We assume that $m, \lambda \in \mathbb{R}$, $m = \text{const} > 0$, and

$$\lambda = \begin{cases} \lambda_1, & y > 0, \\ \lambda_2, & y < 0. \end{cases}$$

Problem BS^∞**.** Find a function $u(x,y)$ such that